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Excise Taxes and Commodity Promotion: Bayesian Retrieval of the Optimum

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This article shows how the solution to the promotion problem—the problem of locating the optimal level of advertising in a downstream market—can be derived simply, empirically, and robustly through the application of some simple calculus and Bayesian econometrics. We derive the complete distribution of the level of promotion that maximizes producer surplus and generate recommendations about *patterns* as well as levels of expenditure that increase net returns. The theory and methods are applied to quarterly series (1978:2–1988:4) on red meats promotion by the Australian Meat and Live-Stock Corporation. A slightly different pattern of expenditure would have profited lamb producers.

Key Words: Bayesian estimation, commodity promotion as an experiment, distribution of the optimum, Taylor-series expansion

This article derives a formal test of the hypothesis that a commodity promotion program is "efficient." By efficient, we mean that the observed expenditure level and the one that maximizes some welfare criterion are the same. The welfare criterion that we adopt is the level of (farmgate) surplus that is generated from sale of the farm commodity. Movements in the commodity price affect this surplus, and the program's objective is to raise this price. This objective is achieved, indirectly, by first raising the corresponding retail price. Given a rise in the retail price, farmgate demand expands, causing the commodity price to rise along the (upward-sloping) commodity supply curve. But, for many reasons, raising retail price is a tricky operation and, in this context, administrators must enact decisions with incomplete information. Uncertainty arises due to vagaries in consumer responsiveness to prices of competing products, health trends, and the effects of the promotion program itself, and this uncertainty raises interesting questions for applied economic analysis. Principal among these is the notion that the desired level of promotion is discovered

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through trial and error. It follows naturally that the optimal level of expenditure to administer is a random variable and, with this notion at hand, a natural way to characterize the "optimum" is to derive an explicit distribution and compare this distribution with the observed distribution of expenditure that the program generates. This is the concept we will pursue in this analysis. To date, the concept has not been exploited in the literature, and thus its analysis makes the current study novel, but also enlightening. It is enlightening because it is capable of identifying not only an optimal level, but also an optimal distribution of expenditure. Hence, the results of our investigation have the potential to generate new insights and, possibly, methodologies aimed at gaining a greater understanding of promotion programs and their worth. Motivating these ideas is the main objective of the exercise.

Relegating details until later, the optimal level of promotion is the level at which a marginal increase in expenditure leaves the commodity price unchanged. Here, "price" refers to the net, per unit return to the farm commodity. It is a post-promotion, post-tax price. The intuition for why it must remain invariant is simple and is articulated graphically in Holloway (1998, p. 189). The intuition follows from the fact that farm profits are monotonically increasing in price. Ceteris paribus, the (post-promotion) price rises in response to an increase in expenditure, but it must also fall in response to an increase in the tax that is required to fund the increase in expenditure. At any optimum, the net impact of the two forces must balance because it is only then that all of the net returns to the program are exhausted. Therefore, at the optimum, marginal adjustments must leave (the post-promotion, post-tax) price unchanged.

Unfortunately, locating the optimum raises some mathematical complications. In an effort to preserve continuity, we relegate much of this detail to an appendix, focusing instead in the body of the article on intuition and a few essential formulae. The basic theory is sketched in the section below, followed by a discussion of econometric implications and a section that motivates the estimation method. The application to Australian data is then reported, followed by the presentation of the empirical model. Conclusions are offered in the final section.

Theory of the Optimum

As outlined in Holloway (1998), the essential features of a promotion program are three-fold. First, promotion output targeted for a downstream market is the result of a production process. Second, the production process creating the output incurs a cost. Third, in order to raise the funds necessary to cover this cost, a tax must be enacted. The purpose of this section is to relate these concepts to a refutable condition that can be used empirically to characterize the optimum. We begin from the intuition already outlined—that, at the optimum, marginal adjustments must leave (the post-promotion, post-tax) price unchanged. The calculus leading to this conclusion is presented in Holloway [1998, equations (2), (3), and (4), p. 191], and is summarized by the following equation:

$$p_\tau \tau_\pi + p_\pi = 0, \tag{1}$$

where p denotes price; τ denotes the tax rate levied for the purpose of raising revenue; subscripts denote partial derivatives; and the endogeneity of the tax rate, $\tau = \tau(\pi)$, is recognized explicitly through the derivative τ_π. Note that equation (1) and the condition $\Delta p = 0$ are equivalent. But, working with equation (1), we are able to construct an equilibrium condition that will prove useful in subsequent developments. This condition evolves naturally from interpreting observed expenditure as the result of experiment, at least locally, in the neighborhood of the equilibrium defined by (1).

Commodity Promotion as an Experiment

Let $\hat{\pi}$ denote the optimal level of promotion measured in the continuum of expenditure, π. In depicting the optimum, our objective is to characterize a probability distribution, say $g(\hat{\pi} \mid \Xi)$. This distribution is important because it summarizes all the relevant information for optimum promotion decision making. Because these decisions are made in a trial-and-error setting, the distribution of the optimum is conditioned by the available data, Ξ, and it follows naturally that Bayesian estimation is appropriate. An estimation strategy that follows logically from interpreting observed expenditure as the result of trial and error, and provides the basis for the Bayesian approach that follows, evolves from two facts. First, because the left-hand side of (1) denotes a partial derivative of an objective function defined over promotion expenditure, it is, by assumption, a function of the control, π. Second, because the optimum is characterized entirely in terms of a derivative, it is no less restrictive to analyze local departures through a Taylor-series expansion of the first-order condition, and interpret sample variance in expenditure as the result of experiment.

Hence, rewriting the first-order condition $f(\pi) \equiv p_\tau \tau_\pi + p_\pi$, noting that (by definition of the optimum) $f(\hat{\pi}) \equiv 0$, and introducing the second-order (slope) parameter $\alpha \equiv f_\pi(\hat{\pi})$, local departures around (1) evolve according to

$$\alpha(\pi - \hat{\pi}) = 0. \tag{2}$$

The expression on the left is exact at the point of departure and holds as an approximation as we move away from the point $\pi = \hat{\pi}$. But, in the neighborhood of $\hat{\pi}$, either (1) or (2) suffices. The second interpretation has the advantage that it generates observations on the distribution of the optimum.

In order to retrieve this distribution, we focus on a set of comparative-static experiments that are relevant to the equilibrium in the marketing system. Relegating details to the appendix, the reduced form consists of the set of price-quantity pairs that are relevant to the commodity in question (price-quantity pairs in the distribution, retailing and processing sectors, and the price and quantity at the farmgate) and one additional equation. This additional equation states simply that the revenue from commodity taxation equals expenditure on the program. Except for this equation, the reduced form is completely general. If we assume that the tax is a per unit levy and that the level of promotion is small in relation to aggregate demand for promotion services, then the equation

$$\kappa\pi = \tau q \tag{3}$$

completes the equilibrium. This is the revenue-equals-cost constraint, where κ denotes the (fixed) per unit cost of promotion, and q denotes the quantity of the commodity exchanged at equilibrium.

It follows from some tedious manipulations (outlined in the appendix), and from the form of equation (3), that reduced-form elasticities depicting farmgate price, quantity, and tax-rate changes are, respectively:

$$\begin{aligned} \psi_{p\pi} &\equiv \frac{\alpha(\pi - \hat{\pi})\pi}{pq}, \\ \psi_{q\pi} &\equiv \frac{\varepsilon\alpha(\pi - \hat{\pi})\pi}{pq}, \\ \psi_{\tau\pi} &\equiv \frac{pq - \varepsilon\alpha(\pi - \hat{\pi})\pi}{pq}. \end{aligned} \tag{4}$$

Here, the expression $\psi_{ij} \equiv \tilde{i}/\tilde{j}$ ($\tilde{k} \equiv \Delta k/k$) defines an elasticity computed from total differentiation of the marketing system, and $\varepsilon \equiv p_q q/p$ defines the (farmgate) supply flexibility computed from partial differentiation of the (farm-commodity) inverse-supply schedule. Consequently, each of the elasticities in (4) are functions of the parameters (α, ε, and $\hat{\pi}$) and the data (p, q, and π), and our task is to retrieve the distributions of the former from observations on the latter. We are primarily interested in the distribution $g(\hat{\pi} \mid \Xi)$.

Bayesian Retrieval of the Optimum

In line with the Taylor-series development underlying equation (2), observations in the time series are interpreted as repeated experiments about the unknown optimum. Given t observations in the time series, the reduced-form equations depicting proportional changes in the commodity price (p), the commodity quantity (q), and the levy (τ) can be written as the linear system:

$$\mathbf{Y} = \mathbf{Z\Pi} + \mathbf{V}, \tag{5}$$

where, in a standard notation (e.g., Drèze and Richard, 1983, p. 519), $\mathbf{Y}_{t\times m} \equiv (\tilde{p}, \tilde{q}, \tilde{\tau})$ is a matrix of observations on the three $\{m = 3\}$ endogenous variables; $\mathbf{Z}_{t\times n} \equiv (\tilde{\pi}; \tilde{\kappa}; \tilde{z}_1, \ldots, \tilde{z}_{n-2})$ is a matrix of observations on $\{n\}$ exogenous variables (respectively, the level of promotion expenditure, the per unit cost of promotion services, and a vector of $\{n - 2\}$ shift variables—for example, prices of substitute goods at retail, disposable income in the retail sector, and prices of nonprimary inputs at other stages of the marketing chain); $\mathbf{\Pi}_{(n \times m)}$ is a matrix of coefficients of the exogenous variables [of which equations (4) comprise a subset]; and the error matrix, $\mathbf{V}_{(t \times m)}$, is assumed to be distributed $N(0, \mathbf{\Omega} \otimes \mathbf{I}_t)$, where $\mathbf{\Omega}_{(m \times m)}$ specifies covariance among the columns of **V**. In the context of estimation, and in view of the (revenue-equals-cost) constraint in (3), at least one column in **V** is a linear combination of two others, implying that the matrix **Ω** is singular. For this reason, the tax-rate equation is excluded from estimation and we focus attention on the commodity price and quantity equations.[1] Hence, only the first two expressions in (4) are estimated. Accordingly, the dimension of **Π** is reduced, and interest centers on isolating and estimating the parameters appearing in the first two rows of $\mathbf{\Pi}_{(n \times (m-1))}$. Conviently, the additive forms of these two expressions permit separation of parameters from data in the first row of coefficients in (5). Accordingly, after separation, $\mathbf{\Pi}_{((n+1) \times (m-1))}$ is

$$(6) \qquad \mathbf{\Pi} \equiv \begin{pmatrix} \alpha & \alpha\varepsilon \\ -\alpha\hat{\pi} & -\alpha\varepsilon\hat{\pi} \\ \psi_{p\kappa} & \psi_{q\kappa} \\ \psi_{pz_1} & \psi_{qz_1} \\ \cdots & \cdots \\ \psi_{pz_{n-2}} & \psi_{qz_{n-2}} \end{pmatrix},$$

and $\mathbf{Z}_{(t \times (n+1))}$ is

$$(7) \qquad \mathbf{Z} \equiv \left(\frac{\pi^2 \tilde{\pi}}{pq};\ \frac{\pi \tilde{\pi}}{pq};\ \tilde{\kappa};\ \tilde{z}_{n-1}, \ldots, \tilde{z}_{n-2} \right).$$

Hence, each of the distributions of interest is now retrievable. From the first two rows in (6), each of the parameters is identified in sequential fashion: α is identified; conditional on α, $\hat{\pi}$ is identified; and conditional on α and $\hat{\pi}$, ε is identified. With these technicalities behind us, an algorithm for retrieving the distributions of interest, $g(\alpha \mid \Xi)$, $g(\hat{\pi} \mid \Xi)$, and $g(\varepsilon \mid \Xi)$, follows easily from some standard results on the natural-conjugate, normal-linear model (Zellner, 1971, pp. 224–240). It is based on four observations. First, given the data, the conditional distribution of the

[1] The analysis is not independent of which equation to delete, and the choice of the tax rate is particularly advantageous. Because the actual farmgate levy is used frequently to fund a variety of ancillary activities (e.g., research and development, market surveys, or other forms of evaluation), the rate implicit to the advertising program is usually unobserved.

covariance matrix, $\mathbf{\Omega}$, is inverse-Wishart. Second, given the data and a draw for the covariance matrix, the distribution of the coefficient matrix, $\mathbf{\Pi}$, is normal. Third, given the draw for the coefficient matrix, each of the parameters of interest (α, $\hat{\pi}$, and ε) is identified from a nonlinear transformation of specific elements in $\mathbf{\Pi}$. Fourth, because it is easy to sample from the distributions in question, this sampling procedure can be repeated a large number of times and the output can be used to plot histograms corresponding to the relevant distributions. Details are presented in the appendix.

Refinements

Prior to discussing the application, four points are noteworthy. First, some flexibility in estimation is available should the researcher have reason to believe that the farm-quantity (rather than the farm-price) equation may lead to more precise estimates of the optimum. However, in applications to time series over short periodicities (such as the present one), it is unlikely that this will be the case. Second, in the extreme case where quantities are reasonably assumed fixed in the sample period, the same procedure can be applied to the (single-equation) estimation of farm-price movements. Two modifications are required in this circumstance. The draw in the first step of the algorithm is inverse-Gamma, and the draw in the second step is multivariate-normal. Third, problems of encountering outliers are a concern whenever the parameters in the denominator of the computations in the second step are located near zero. In situations where the potential number of outliers is considerable, or their numerical values are large, or both, it is recommended that these observations be discarded from the sample. The problem of constructing a (subjective) criterion for stopping can be handled by discarding, simultaneously, the endpoints of the distributions until the base of each of the frequency increments contains at least one observation. Finally, in situations in which the curvature of the objective function lies in question, probabilities that parameter α takes a particular sign are easily constructed from the output of the sample. Moreover, constraints such as $\alpha \leq 1$, $\hat{\pi} \geq 0$, or $\varepsilon \geq 0$ (as predicated by the theory) are easily imposed by modifying the draws in the second step to be truncated-normal.[2] This can be achieved easily through a simple accept-reject mechanism, whereby draws are continued until a draw from the desired region is obtained. An efficient one-to-one draw (using the probability-integral transform) is outlined in Chib (1992).

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[2] As a colleague points out, it is worth emphasizing some subtle relationships that exist between the two constraints $\alpha \leq 0$ and $\hat{\pi} \geq 0$. There are two. First, the restriction $\alpha \leq 0$ is a necessary condition for the existence of a local optimum, for which the restriction $\hat{\pi} \geq 0$ characterizes a feasible set of configurations for the program. Note, of course, that point mass at zero (in other words, the outcome $\hat{\pi} = 0$) is feasible and is enlightening, but only in the event that $\alpha \leq 0$. Second, as shown in the appendix, the two constraints are not independent. In fact, due to the relationship between α and $\hat{\pi}$ in (6), the sampling procedure that generates the distribution $\hat{\pi} \geq 0$ is affected fundamentally by the restriction $\alpha \leq 0$.

Application

The theory and procedures are applied to a consistent set of quarterly observations (1977:2–1988:4) on red meats advertising by the Australian Meat and Live-Stock Corporation (AMLC). The AMLC has, since the beginning of 1977, conducted a multi-media (radio, television, and newspaper) campaign in an effort to stem declines of domestic consumption of red meats (predominantly beef and lamb). Declines in consumption of both commodities accelerated significantly in the early 1970s, and lamb consumers in particular have shown little tendency of redirecting this trend. Application of the theory to the Australian lamb market requires three features of the marketing environment to be considered prior to estimation. A first issue concerns the significance of international trade in domestic price determination; a second concerns the timing of promotion expenditure in relation to the timing of transactions in the domestic retail market and the relationship, in turn, between this transaction and the sale of the raw commodity; third, because the data are quarterly observations, an appropriate specification of quarterly supply response must be selected.

Because the data periodicity is significantly less than the time considered to adjust effectively supplies of live animals, the application is undertaken based on the assumption that supply is fixed [in other words, that parameter ε in equation (6) tends to infinity]. Turning to the appropriate lag specification in the promotion-retail-farm sequence, a grid search is conducted in order to locate the lag combination that maximizes posterior probability across combinations, up to a maximum of eight quarters. The procedure amounts to comparing the (posterior-means) residual sums of squares across a ($9 \times 9 = 81$ point) grid. The optimal promotion-to-retail response is four quarters, and the optimal retail-to-farm response is six quarters.[3]

Finally, in an effort to account for trade effects, the structure of the export lamb market is considered. Key export destinations for Australian lamb are the major coastal centers in the eastern and western United States. Therefore, preliminary estimation was conducted on a time series that includes demand- and trade-shift variables that are relevant to these markets. Specifically, the impacts of the U.S.-Australian exchange rate, U.S. prices of substitute meats (beef, pork, chicken), and U.S. meat expenditure are considered. Dependence of movements in Australian farm prices on each of these variables is negligible, implying that export conditions are not strong determinants of domestic equilibrium in the Australian lamb market. Consequently, the remaining discussion focuses on domestic price determination with fixed supplies of live animals and four- and six-period lags, respectively, in promotion-to-retail and retail-to-farm linkages.

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[3] A reviewer questions the statistical properties of the test procedure, but these properties are actually well known. Due to the fact that the regressor matrix, $\mathbf{Z}$, changes across the 81-point grid, the evaluations reduce simply to comparisons of posterior probabilities across each of the 81 (nonnested) regression models. The formulae for their comparison are developed in Zellner [1971, pp. 306–309, equations (10.33)–(10.48)].

The Empirical Model

Prior to estimation, the data are converted to logarithmic differences. That is, we replace $\tilde{k} \equiv \Delta k/k$ [as defined below (4)] with its empirical counterpart, $\tilde{k} \equiv \log(k_t) - \log(k_{t-1})$. In the estimation, we restrict attention to the relationship between farm-price movements and 12 regressors. Respectively, these 12 regressors are identified as follows [see equation (7)]:

1. promotion expenditure ($Aus 000) squared times the proportional change in expenditure, normalized by commodity revenue (henceforth, "promotion squared");
2. promotion expenditure ($Aus 000) times the proportional change in expenditure, normalized by commodity revenue (henceforth, simply denoted "promotion") ($Aus 000);
3. an index of per unit promotion costs (the aggregate promotion costs index for Australia, as in Ball and Dewbre, 1989, table 12);

4–6. domestic retail prices of beef, pork, and chicken (cents per kilogram);

7. retail meat expenditure ($Aus 000);
8. wholesale lamb quantities (metric tons, assumed to be exogenous); and

9–12. four quarterly dummy variables (the regression excludes a constant).

The source for the promotion data is Ball and Dewbre (1989, tables 11–13). For the remaining variables, various publications of the Australian Bureau of Agricultural and Resource Economics (ABARE) are used.

Figure 1 presents nominal expenditure ($Aus 000) on red meats promotion by the AMLC. The upper panel presents the quarterly pattern over the time series (1978:2–1988:4), and the lower panel depicts the empirical distribution of the quarterly expenditure across percentile increments in the range $Aus 100 (1978:3) to $Aus 748,000 (1988:3). The numbers at the base of the lower panel are, respectively, from left to right, the minimum, the mean, and the maximum expenditure in the time series. The question we now consider is whether this distribution is "optimal."

Results

Table 1 reports numerical results and figure 2 reports plots of key distributions obtained from single-equation, unconstrained estimation with a sample size set at $S = 10{,}000$. There are 194 outliers, leaving a total of 9,806 remaining observations. The outliers are defined according to the previous convention whereby the outermost observations in the sample are discarded until each 1% increment in the domain of the distribution contains at least one observation. The two distributions appearing in figure 2 are the second-order derivative of the objective function (α)

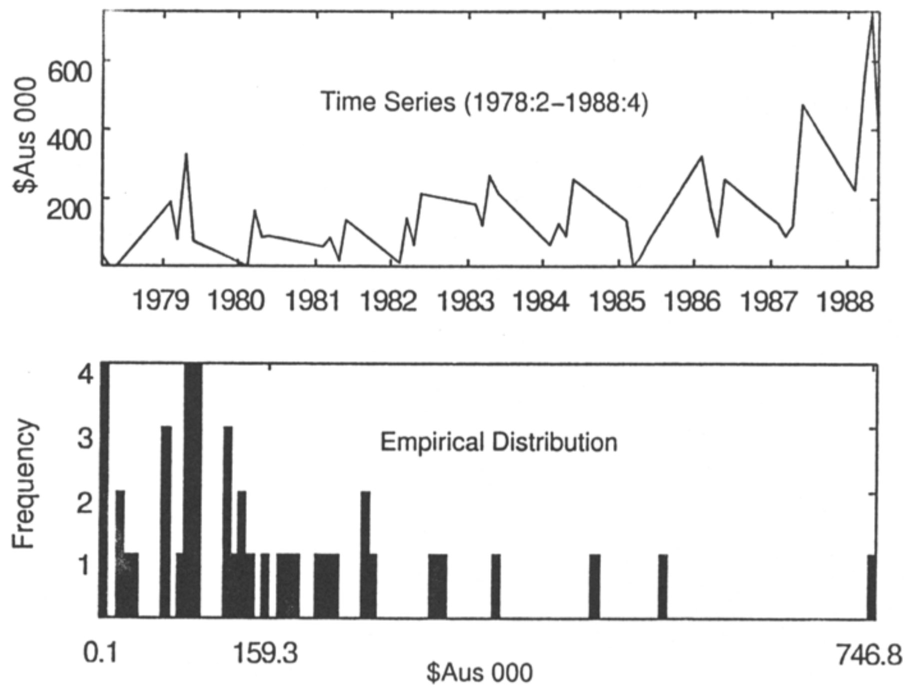

Figure 1. AMLC promotion expenditure (1973:2–1988:4)

and the optimal level of promotion expenditure ($\hat{\pi}$). The numbers on the vertical axis represent the sampling frequencies of percentile increments, and the numbers on the horizontal axis are, respectively, from left to right, the minimum, the mean, and the maximum of the sample observations remaining after removing outliers. The distribution of the slope parameter (α)—which is normal—is dispersed, but has a mode at a negative value (-0.002); a significant proportion of its mass lies in the negative domain. The distribution of the optimum ($\hat{\pi}$)—which is not normal—is peaked with long tails and appears to be centered about its mean ($Aus 242,000).

Other measures of interest are summarized in table 1. Generally speaking, the farm price responds positively to the price of pork, negatively to lamb quantity and to the first-quarter dummy variable, and is not significantly affected by the remaining regressors. These are reasonable results in the context of the marketing system for lamb, and they are somewhat expected. But their most significant implication is their prediction about optimal promotion expenditure. The mean of the distribution of the optimum ($Aus 242,000) exceeds the mean of the empirical distribution ($Aus 159,000) by a considerable margin—about 52% of the mean of the empirical distribution. Thus, with a few caveats (discussed below), lamb producers would have benefitted from an overall increase in promotion expenditure.

Table 1. Unrestricted and Restricted Estimates of Lamb Price Determination (1978:2–1988:4)

Description	Unrestricted Estimates	Restricted Estimates
Elasticities of Farm Price with Respect to:		
Promotion squared	−0.002 (0.002)	−0.003 (0.002)
Promotion	0.375 (0.918)	0.910 (0.637)
Per unit promotion costs	0.145 (0.178)	0.154 (0.172)
Price of beef	−0.249 (0.638)	−0.287 (0.628)
Price of chicken	−0.140 (0.680)	−0.090 (0.661)
Price of pork	1.884 (0.753)	1.986 (0.743)
Meat expenditure	0.164 (0.343)	0.199 (0.330)
Lamb quantity	−1.213 (0.185)	−1.185 (0.181)
First-quarter dummy	−0.177 (0.039)	−0.177 (0.038)
Second-quarter dummy	0.048 (0.037	0.051 (0.037)
Third-quarter dummy	−0.008 (0.036)	−0.016 (0.035)
Fourth-quarter dummy	0.007 (0.056)	0.004 (0.055)
Summary Statistics:		
Square root of the variance	0.071 (0.010)	0.071 (0.011)
Optimal promotion expenditure ($Aus 000)	242.275 (869.372)	315.760 (187.252)
Probability $\alpha \geq 0$		0.181 (0.385)
Probability $\hat{\pi} \leq 0$		0.343 (0.475)
Sample size	9,806	9,922

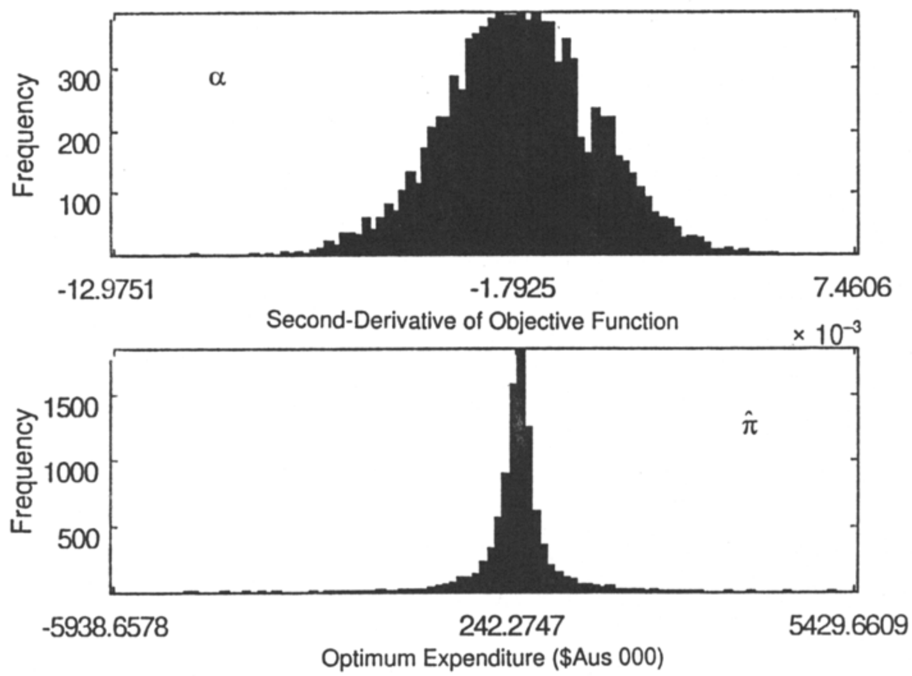

Figure 2. Posterior distributions: Single-equation, unrestricted estimation

Given the locations of the unrestricted distributions, it is likely that the theoretical constraints $\alpha \leq 0$ and $\hat{\pi} \geq 0$ are compatible with the data. As indicated previously, these restrictions can be tested conveniently by truncating the draws of the first two coefficients in the price equation. The results of the restricted estimation are presented in the second numeric column of table 1. There are 78 outliers, leaving a sample size of 9,922 observations. Neither the means nor the (numerical) standard errors of the estimates appear to be greatly affected by the restrictions. Moreover, the probabilities that $\alpha > 0$ and $\hat{\pi} < 0$ are 0.18 and 0.34 (with standard errors 0.38 and 0.48), respectively. Thus, we conclude that the data are reasonably compatible with the theoretical restrictions.

Figure 3 presents reports of the distributions of the slope term (α) and the optimum ($\hat{\pi}$) obtained from restricted estimation. The values at the bases of the two panels are, respectively, from left to right, the minimum, the mean, and the maximum of the observations in the sample. The left-wise skewness in the distribution of α is obvious and, compared to the unrestricted estimate, its mean has declined somewhat (to around -0.003), which is to be expected. The distribution of the optimum ($\hat{\pi}$) also exhibits skewness, but in this case the skewness is rightward.

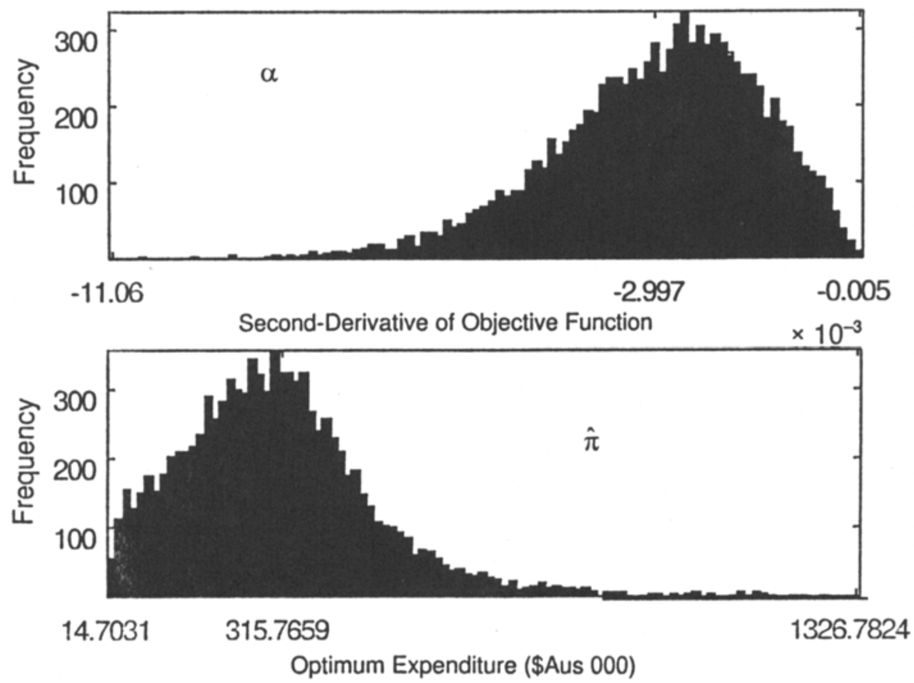

Figure 3. Posterior distributions: Single-equation, restricted estimation

Significantly, the mean of the distribution ($Aus 316,000) represents almost a 100% increase over the mean of the observed distribution ($Aus 159,000). Thus, we conclude once again that an increase in mean expenditure would have profited lamb producers.

These conclusions are supported in different terms by the graphic in figure 4. The figure compares the empirical distribution of promotion expenditure (upper panel), with the distribution of the optimum obtained from unrestricted estimation (center panel), and the distribution of the optimum obtained from restricted estimation (lower panel). In order to aid visual inspection, both of the posterior distributions are truncated to an interval that is approximately twice the length of the domain of the empirical distribution (approximately ± $Aus 1.5 million). The tick marks signify increments of $Aus 500,000, but also report the means of the respective distributions. The graphic clearly reveals differences in expenditure that would improve lamb producer profits. When the theoretical constraints $\alpha \leq 0$ and $\hat{\pi} \geq 0$ are ignored, the distribution of the optimum is quite normal, and is peaked; but when the constraints are imposed, the distribution resembles that of an inverse-Gamma distribution with low degrees of freedom. In both cases, however, evidence of an

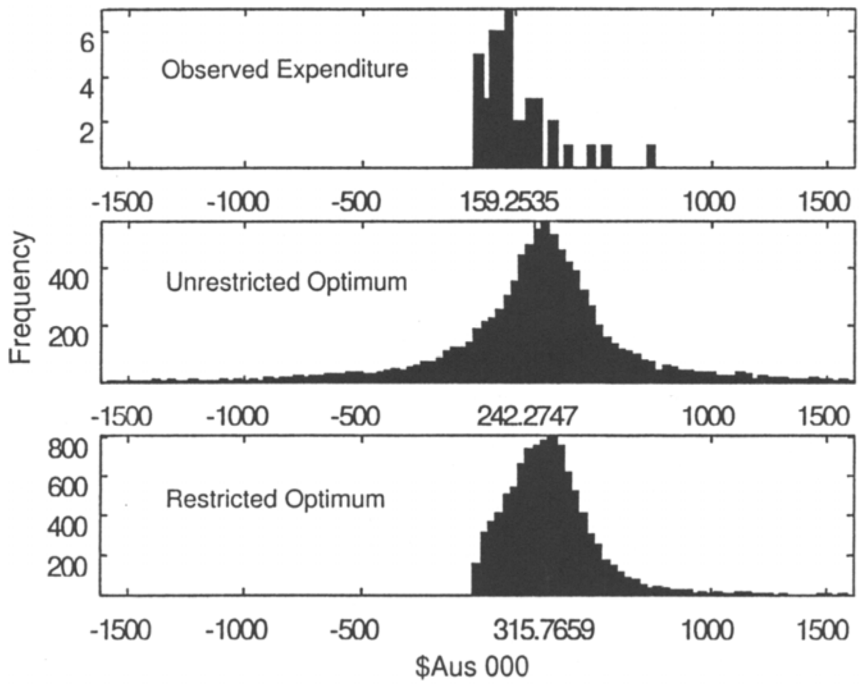

Figure 4. Observed, unrestricted-optimal, and restricted-optimal distributions

overallocation in the lower percentiles of the empirical distribution is apparent. The average level of observed expenditure lies below the means of the distributions of the two optima, but the patterns of expenditure are, perhaps, more informative. The patterns in the lower panels suggest that a small but significant frequency of expenditure at levels twice the maximum of the observed expenditure would be beneficial. Adjusting promotion expenditure in this manner would have profited Australian lamb producers.

Summary and Policy Conclusions

This article extends the logic of a recent contribution to show how inferences about the optimal level of promotion can be retrieved empirically, conveniently, and robustly using Bayesian techniques. The approach is attractive because it yields enlightening results with fundamental implications, not only for the mean, but for the entire distribution of promotion expenditure. In this respect, the policy conclusions stemming from our findings warrant closer scrutiny.

The principal rationale for the introduction and use of commodity promotion schemes is that they provide an effective means of raising farm income. A logical consequence of this assumption is a question surrounding the level of promotion expenditures that should be applied, taking account of the farm-price impacts of retail-demand shifts, changes in costs of production in the marketing sector, and changes in supply conditions in farming itself. This point is worth emphasizing; prediction about the appropriate levels of promotion expenditure is the overarching measurement issue confronting executives of commodity-promotion schemes. In confronting this issue directly, the methodology employed here provides clear and precise answers to this policy question. This is the principal objective addressed in the paper. However, unlike the bulk of previous contributions that have sought point-estimate predictions, the ones contained here are more comprehensive because they generate predictions about complete distributions of expenditure. In short, given the assumption that the optimum level of expenditure is a random variable, an optimal distribution (pattern of expenditure) exists. This article has shown how the existence of this distribution is motivated, how to characterize it mathematically, and how to derive it empirically in order to generate clear, concise policy conclusions.

In the present setting, the main policy conclusions are twofold. The first is that the AMLC, during the period 1978:2–1988:4, should have increased, on average, its red meats promotion allocations and that this increase should have been targeted to a one-and-one-half to two-fold increase over the level of observed expenditure. The second conclusion concerns the patterns of reallocation. First, a greater spread, with more frequency in the right tail of the distribution, and, second, a concentration (a series of repeated allocations) around $Aus 316,000 would have maximized the overall return to the scheme, enhanced producer profits, and maximized returns to the farmgate levy required to raise program revenues. These are the main conclusions one can draw from the analysis; they are, arguably, the most important conclusions for promotion policy, and consequently are the ones we have focused on in the present contribution.

Notwithstanding this focus, a host of other important policy questions remain. A few are addressed above, whereas others are available from modest extensions of the present methodology. The addressed issues are the validity of maintained hypotheses including the existence of a maximum (as opposed to a minimum) and the positivity of the distribution of the optimum, given its existence. As this analysis has shown, these questions—which are fundamental to the optimization hypothesis—are easily refuted or confirmed empirically.

Of the remaining questions, interest centers on the (comparative-static) effects of (optimal) responses in promotion to changes in the economic environment, such as shifts in retail demand, shifts in costs of production in the marketing sector, and shifts in supply conditions in the farm sector itself. Magnitudes of optimal responses to changes in, say, prices of retail-competing products (in this case, beef and pork), changes in expenditures on particular commodity groups (presently, meats), changes in food-marketing costs (presently approximated by labor wage rates), changes in supply conditions (represented by wholesale lamb supply), or changes in commodity promotion costs themselves are easily derived (either in point-estimate or distribution form) by iterating the estimation procedure outlined in the appendix. These estimates are important because they yield information regarding the appropriate response by administrators to changes in the economic environment. In short, a variety of policy information, additional to the central questions presently considered, are obtainable from modest extensions of the methodology.

Future work should aim to exploit this knowledge while relaxing simultaneously some of the stringent assumptions employed currently, including a single-lag response to promotion (e.g., Kinnucan, Chang, and Venkateswaran, 1993), a single-commodity marketing channel (e.g., Kinnucan, 1996), a single-promotion medium, but most importantly that the goal of promotion is to maximize static, nonstochastic producer surplus.

References

Ball, K., and J. Dewbre. (1989). "An analysis of the returns to generic advertising of beef, lamb, and pork." Discussion Paper No. 89.4, Australian Bureau of Agricultural and Resource Economics. Canberra, Australia: Government Publishing Service.

Bauwens, L. (1984). *Bayesian Full-Information Analysis of Simultaneous Equation Models Using Integration by Monte Carlo.* Lecture Notes in Economics and Mathematical Systems No. 232. Berlin: Springer-Verlag.

Chib, S. (1992). "Bayes inference in the tobit censored regression model." *Journal of Econometrics* 52, 79–99.

Drèze, J. H., and J. F. Richard. (1983). "Bayesian analysis of simultaneous equation systems." In Z. Griliches and M. D. Intriligator (eds.), *Handbook of Econometrics*, Vol. 1 (pp. 517–596). Amsterdam: North-Holland.

Gelman, A., J. B. Carlin, H. S. Stern, and D. B. Rubin. (1997). *Bayesian Data Analysis.* New York: Chapman and Hall.

Holloway, G. J. (1998). "Excise taxes and commodity promotion: A diagrammatic motivation." *Journal of Agribusiness* 16(2), 187–192.

Kinnucan, H. W. (1996). "A note on measuring returns to generic advertising in interrelated markets." *Journal of Agricultural Economics* 47, 261–267.

Kinnucan, H. W., H.-S. Chang, and M. Venkateswaran. (1993). "Generic advertising wearout." *Review of Marketing and Agricultural Economics* 61, 401–415.

Zellner, A. (1971). *An Introduction to Bayesian Inference in Econometrics.* New York: Wiley, Wiley Classics Library edition.

Appendix

In this appendix we present details underlying some of the essential formulae appearing in the text. We first transform equation (1) into an expression that is more amenable to empirical analysis. In this endeavor, we make use of some additional notation and three results that follow from basic assumptions in Holloway (1998, p. 188). First, given equality between the variable costs of promotion, $C(\pi)$, and the revenue raised to cover them, $R(\tau, \pi)$, the rate of change in the tax rate that arises in response to a change in the level of promotion is

$$\text{(A1)} \qquad \tau_\pi = \frac{C_\pi - R_\pi}{R_\tau}.$$

Second, assuming the usual situation in which the promotion program's demand for resources is small in relation to total demand, free entry and exit into the promotion industry ensures that the promotion-costs function exhibits locally constant returns to scale. Thus, assume that $C(\pi) \equiv \kappa\pi$, where κ denotes the per unit cost of promotion output. Accordingly, the first term in the numerator in (A1) is

$$\text{(A2)} \qquad C_\pi = \kappa.$$

Without loss of generality, assume that the tax rate τ denotes a per unit (excise) rate of taxation. Accordingly, with $q(\tau, \pi)$ denoting equilibrium quantity, receipts from taxation are $R(\tau, \pi) \equiv \tau q(\tau, \pi)$. Hence, the second term in the numerator in (A1) is

$$\text{(A3)} \qquad R_\pi = \tau q_\pi,$$

and the term in the denominator is

$$\text{(A4)} \qquad R_\tau = q + \tau q_\tau.$$

Third, consider the (inverse) farm-commodity supply schedule depicting producers' real opportunity costs of production, $p = S(q)$, $S_q > 0$. In equilibrium, where both price and quantity are functions of the policy instruments [that is, $p = p(\tau, \pi)$ and $q = q(\tau, \pi)$], we have $p_\tau = S_q q_\tau$ when the tax rate changes, and $p_\pi = S_q q_\pi$ when promotion adjusts. Consequently, when both the tax rate and the level of promotion are adjusted simultaneously,

$$\text{(A5)} \qquad p_\tau q_\pi = p_\pi q_\tau.$$

Equations (A1)–(A5) can now be applied to text equation (1) in order to derive a condition that is more amenable to analysis. Substituting, in turn, equations (A1), (A2), (A3), and (A4) into equation (1) and rearranging, we have

$$p_\tau(\kappa - \tau q_\pi) + p_\pi(q + \tau q_\tau) = 0. \tag{A6}$$

Using (A5),

$$p_\tau \kappa + p_\pi q = 0. \tag{A7}$$

Applying text equation (3),

$$p_\tau \tau + p_\pi \pi = 0. \tag{A8}$$

Finally, normalizing on $p \neq 0$, we have

$$E_{p\tau} + E_{p\pi} = 0, \tag{A9}$$

where $E_{ij} \equiv (\partial i / \partial j)(j / i)$ denotes the elasticity of variable i with respect to variable j. Equation (A9) provides the key link between the theoretical optimum and its empirical measurement. Intuitively, because taxation leads to losses in producer surplus and because promotion (presumably) effects benefits, equation (A9) defines the familiar rule that, at the optimum, the marginal (producer) benefit from promotion, $E_{p\pi}$, must equal the marginal cost, $-E_{p\tau}$, or that the net marginal benefits of the commodity promotion scheme must be exhausted. However, a second and more important feature of (A9) is that it is, in principle, observable given data on prices and quantities and the levels of the relevant instruments, τ and π.

When interest centers on empirically testing condition (A9), a test procedure evolves in three steps. First, we consider the set of quasi-reduced forms that characterize equilibrium in the food channel. Second, we impose on these equations the identity implied by the cost-equals-revenue constraint. Third, we identify from the corresponding set of reduced-form equations a set of restrictions that are equivalent to, but more informative than, the one in (A9). These concepts are formalized as follows. Farmgate equilibrium in the presence of promotion is

$$\begin{aligned} p &= f^p(\tau;\ \pi;\ z_1, \ldots, z_{n-2}), \\ q &= f^q(\tau;\ \pi;\ z_1, \ldots, z_{n-2}), \end{aligned} \tag{A10}$$

where $f^p(\cdot)$ and $f^q(\cdot)$ denote the (quasi) reduced forms depicting price and quantity, and $\mathbf{z} \equiv (z_1, \ldots, z_{n-2})'$ represents the effects of other variables on the equilibrium. Recall that the tax rate and the level of promotion effort are related by the budget constraint in text equation (3). In this case, (A10) and (3), together, comprise a three-equation structural system, with endogenous variables p, q, and either the tax rate, τ, or the level of promotion effort, π, as endogenous variables. Displacing the system, expressing the resulting derivatives in proportional-change terms, we obtain the following:

(A11) $$\tilde{p} = E_{p\tau}\tilde{\tau} + E_{p\pi}\tilde{\pi} + E_{pz_1}\tilde{z}_1, \ldots, E_{pz_{n-2}}\tilde{z}_{n-2},$$

$$\tilde{q} = E_{q\tau}\tilde{\tau} + E_{q\pi}\tilde{\pi} + E_{qz_1}\tilde{z}_1, \ldots, E_{qz_{n-2}}\tilde{z}_{n-2},$$

and, from text equation (3),

(A12) $$\tilde{\kappa} + \tilde{\pi} = \tilde{\tau} + \tilde{q},$$

where $\tilde{v} \equiv \Delta v / v$ denotes proportional change in variable v. With τ endogenous, the system is

(A13) $$\mathbf{B}\mathbf{y} = \mathbf{\Gamma}\mathbf{x},$$

where $\mathbf{y}_{(m\times 1)} \equiv (\tilde{p}, \tilde{q}, \tilde{\tau})'$ denotes an m-vector of movements in the endogenous variables, $\mathbf{x}_{(n\times 1)} \equiv (\tilde{\pi}; \tilde{\kappa}; \tilde{z}_1, \ldots, \tilde{z}_{n-2})'$ denotes an n-vector of movements in the exogenous variables, and the coefficient matrices $\mathbf{B}_{(m\times m)}$ and $\mathbf{\Gamma}_{(m\times n)}$ are defined, respectively, as follows:

(A14) $$\mathbf{B} \equiv \begin{pmatrix} 1 & -E_{p\tau} \\ 1 & -E_{q\tau} \\ 1 & 1 \end{pmatrix}$$

and

(A15) $$\mathbf{\Gamma} \equiv \begin{pmatrix} E_{p\pi} & E_{pz_1} & \cdots & E_{pz_{n-2}} \\ E_{q\pi} & E_{qz_1} & \cdots & E_{qz_{n-2}} \\ 1 & 1 & \ldots & 0 \end{pmatrix}.$$

The solution in (A13) is

(A16) $$\mathbf{y} = \mathbf{\Phi}\mathbf{x},$$

where $\mathbf{\Phi}_{(m\times n)}$, defined as

(A17) $$\mathbf{\Phi} \equiv \begin{pmatrix} \Psi_{p\pi} & \Psi_{p\kappa} & \Psi_{pz_1} & \cdots & \Psi_{pz_{n-2}} \\ \Psi_{q\pi} & \Psi_{q\kappa} & \Psi_{qz_1} & \cdots & \Psi_{qz_{n-2}} \\ \Psi_{\tau\pi} & \Psi_{\tau\kappa} & \Psi_{\tau z_1} & \cdots & \Psi_{\tau z_{n-2}} \end{pmatrix},$$

satisfies $\mathbf{\Phi} = \mathbf{B}^{-1}\mathbf{\Gamma}$. Finally, focusing attention on (A16), applying Cramer's rule in (A13), and using (A6), we have the following as the relationship between the structural- and reduced-form parameters:

(A18)
$$\psi_{p\pi} \equiv \frac{E_{p\tau} + E_{p\pi}}{|\mathbf{B}|},$$
$$\psi_{q\pi} \equiv \frac{E_{q\tau} + E_{q\pi}}{|\mathbf{B}|},$$
$$\psi_{\tau\pi} \equiv \frac{1 - E_{q\pi}}{|\mathbf{B}|},$$

where $|\mathbf{B}| \equiv 1 + E_{q\tau} \neq 0$.

Because they relate closely to the optimality condition (A9), relations (A18) provide the basis for empirical examination of the optimality hypothesis. The natural strategy is to test the (point) null hypothesis that the parameter $\psi_{p\pi}$ is zero. This condition, we note, is equivalent to the condition that the parameter $\psi_{q\pi}$ is zero because the elasticities in the numerators in (A18) are monotonic transformations of each other, with the supply flexibility $\varepsilon \geq 0$ acting as the scale factor. Nevertheless, having tested these hypotheses, the problem arises in inferring difference between optimal and observed levels of promotion expenditure. A solution to this problem is proposed below.

Text equations (5), (6), and (7) provide the essential observations, and an estimating algorithm is derived in three consecutive steps. First, a relationship is derived between a parameter representing the desired level of promotion expenditure and the empirically observed data. Second, the (posterior) distribution of the optimal level of program expenditure is identified from a nonlinear transformation of the reduced form. Third, the posterior distribution thus derived is compared with the observed frequencies of actual expenditure, and recommendations are made about changes in expenditure levels.

Given the distributional assumption on the error matrix, the joint posterior for the covariance matrix ($\mathbf{\Omega}$) and the regression parameters ($\mathbf{\Pi}$), conditional on the data ($\mathbf{\Xi}$), is normal-inverse-Wishart (Zellner, 1971, pp. 224–240). Although this result is standard in the multivariate-normal, linear setting, this point is worth emphasizing because it is fundamental to the sampling procedure. Put another way, the joint posterior has the component, conditional forms:

(A19)
$$\mathbf{\Omega} \mid \mathbf{\Xi} \sim \text{inverse-Wishart}\,(\mathbf{W}_*, \nu_*),$$
$$\mathbf{\Pi} \mid \mathbf{\Omega}, \mathbf{\Xi} \sim \text{matricvariate-normal}\,(\mathbf{\Pi}_*, \mathbf{\Omega} \otimes \mathbf{M}_*^{-1}),$$

where the definitions of the (data-plus-prior) parameters ($\mathbf{W}_*$, ν_*, $\mathbf{\Pi}_*$, and $\mathbf{M}_*$) are presented in Drèze and Richard (1983, pp. 539–541). Given the relationships between parameters α, $\hat{\pi}$, and ε, and parameter matrices $\mathbf{\Pi}$ and $\mathbf{\Omega}$ in text equation (7), the normal-inverse-Wishart form can be used to simulate draws from the distributions $g(\alpha \mid \Xi)$, $g(\hat{\pi} \mid \Xi)$, and $g(\varepsilon \mid \Xi)$. Denoting the element in the *i*th row and the *j*th column of $\mathbf{\Pi}$ by Π_{ij}, the sampling procedure that simulates draws from the relevant distributions is implemented as follows:

STEP 1. Generate a draw $\mathbf{\Omega}_{(s)}$ from the inverse-Wishart $(\mathbf{W}_*,\ v_*)$.

STEP 2. Generate a draw $\mathbf{\Pi}_{(s)}$ from the matricvariate-normal $(\mathbf{\Pi}_*,\ \mathbf{\Omega}_{(s)} \otimes \mathbf{M}_*^{-1})$, where $\mathbf{\Omega}_{(s)}$ is the draw in step 1.

STEP 3. Compute $\hat{\pi}_{(s)} = -\Pi_{21(s)}/\Pi_{11(s)}$ and $\varepsilon_{(s)} = \Pi_{12(s)}/\Pi_{11(s)}$, where $\Pi_{11(s)}$, $\Pi_{12(s)}$, and $\Pi_{21(s)}$ are elements of $\mathbf{\Pi}_{(s)}$ drawn in step 2.

STEP 4. Repeat steps 1 through 3 a large number of times, S.

Let $\{\alpha_{(s)},\ s = 1, 2, \ldots, S\}$, $\{\hat{\pi}_{(s)},\ s = 1, 2, \ldots, S\}$, and $\{\varepsilon_{(s)},\ s = 1, 2, \ldots, S\}$ denote a sequence of draws so obtained. With S set large, this sequence provides a good approximation to the (post-data) marginal distributions $g(\alpha \mid \Xi)$, $g(\hat{\pi} \mid \Xi)$, and $g(\varepsilon \mid \Xi)$, and their Monte Carlo approximations are retrieved by plotting histograms over increments in their domains. Efficient algorithms for drawing from matricvariate-normal and inverse-Wishart distributions are outlined in Bauwens (1984) and in Gelman et al. (1997).